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LETTER TO THE EDITOR

Optical bistability in dispersive and absorptive media

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Abstract. The relevance of dispersion in optical bistability is outlined. The results are compared with those for absorptive bistability, and a connection with experiments is discussed.

Most experiments on optical bistability deal with dispersive systems such as Na gas (Gibbs *et al* 1976, 1978) and ruby (Venkatesan and McCall 1977), or with crystals showing electro-optical nonlinear effects (Smith *et al* 1978, Garmire *et al* 1978). The theory, however, is mainly concerned with the transmission properties of absorbers (McCall 1974, Bonifacio and Lugiato 1976, 1978a, b, Narducci *et al* 1978, Carmichael and Walls 1978, Willis 1977). A theory for the dispersive case exists only for a medium showing third-order nonlinearity (Marburger and Felber 1978).

The aim of this letter is to relate the different experimental and theoretical approaches by emphasising the role of nonlinear dispersion in optical bistability. For the sake of simplicity, we shall discuss in detail the transmission properties of a two-level atomic system only. Our conclusions, however, are valid for more general dispersive systems, as we shall discuss later. Some preliminary results on dispersive bistability in two-level atomic systems have been reported by Bonifacio and Lugiato (1978c). We start from an N two-level atomic system interacting in a cavity with an electromagnetic field $E(z, t)$. The mirrors of the cavity are characterised by a transmittivity T and a reflectivity $R \equiv 1 - T$. The Maxwell–Bloch equations in the dipole, rotating wave and slowly varying envelope approximations read

$$S^{\pm} = \mp 2\mathrm{i}\lambda E^{\pm} S_3 - (\gamma_{\perp} \mp \mathrm{i}\Delta\omega) S^{\pm} \tag{1a}$$

$$S_3 = \mathrm{i}\lambda (E^{+} S^{-} - E^{-} S^{+}) - \gamma_{\parallel}(S_3 - N/2) \tag{1b}$$

$$\partial E^{\pm}/\partial t + C\, \partial E^{\pm}/\partial x = \pm \mathrm{i}\lambda S^{\pm} \tag{1c}$$

where $S^{\pm}$ are the complex components of the polarisation, $S_3 = (N_1 - N_2)/2$ is the inversion, $E^{\pm}$ are the positive and negative frequency components of the electromagnetic field, $\gamma_{\parallel}$, $\gamma_{\perp}$ are the inverse longitudinal and transverse atomic relaxation times, λ is the dipolar coupling constant, and $\Delta\omega$ is the difference between atomic transition and field carrier frequencies. At time $t = 0$, the atoms are in their ground state, i.e. $S_3 = N/2$.

There exists a well defined limit in which equations (1) reduce formally to that for a monomode field interacting with a two-level system. This limit has been discussed by Bonifacio and Lugiato (1978b) for a ring cavity and by Meystre (1978) for a Fabry–Pérot cavity. We sketch this procedure for (1) in the stationary case. First integrate (1)

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over the cavity length L and use the factorisation *ansatz*

$$\langle AB\rangle \equiv L^{-1}\int_0^L \mathrm{d}z A(z)B(z) = \langle A\rangle\langle B\rangle \tag{2}$$

where A, B can be field as well as atomic variables. The *ansatz* (2) is justified in the limit (Bonifacio and Lugiato 1978a, b, c)

$$\alpha L \to 0, \qquad T \to 0 \tag{3a}$$

$$\alpha L/T = \text{const} \tag{3b}$$

where $\alpha = \lambda^2 N/c\gamma_\perp$ is the absorption constant, and c is the velocity of light. Using (2) and (3) from (1), we obtain in the limit $t \to \infty$

$$(\gamma_\perp \mp \mathrm{i}\Delta\omega)S^\pm = \mp 2\mathrm{i}\lambda T^{-1/2}E^\pm S_3 \tag{4a}$$

$$\gamma_\parallel(S_3 - N/2) = \mathrm{i}\lambda T^{-1/2}(E^+S^- - E^-S^+) \tag{4b}$$

$$\kappa(E_\mathrm{T}^\pm - E_\mathrm{I}^\pm) = \pm\mathrm{i}\lambda S^\pm \tag{4c}$$

where $\kappa = cT^{1/2}/L$. In deriving (4) we have assumed boundary conditions appropriate to a ring cavity, namely

$$E(L) = E_\mathrm{T}T^{-1/2} \tag{5a}$$

$$E(0) = T^{1/2}E_\mathrm{T} + RE(L) \tag{5b}$$

where E_I and E_T are the incoming and transmitted field respectively. From (4) we obtain the following relation between incoming and transmitted field:

$$y = x + 2C_1x/(1+|x|^2) - 2\mathrm{i}C_2x/(1+|x|^2) \tag{6}$$

where x and y are the normalised amplitudes, namely

$$x = \frac{1}{T^{1/2}}\frac{2\lambda E_\mathrm{T}}{(\gamma_\perp^2+\Delta\omega^2)^{1/2}}\left(\frac{\gamma_\perp}{\gamma_\parallel}\right)^{1/2} \tag{7a}$$

$$y = \frac{1}{T^{1/2}}\frac{2\lambda E_\mathrm{I}}{(\gamma_\perp^2+\Delta\omega^2)^{1/2}}\left(\frac{\gamma_\perp}{\gamma_\parallel}\right)^{1/2}. \tag{7b}$$

The normalisation factor $(\gamma_\perp T/\gamma_\parallel)(\gamma_\perp^2+\Delta\omega^2)/(2\lambda)^2$ corresponds to the saturation intensity in a dispersive medium. In (6) we have furthermore introduced the constants

$$2C_1 = \lambda^2\gamma_\perp N/T^{1/2}\kappa(\gamma_\perp^2+\Delta\omega^2) \tag{8a}$$

and

$$2C_2 = \lambda^2\Delta\omega N/T^{1/2}\kappa(\gamma_\perp^2+\Delta\omega^2). \tag{8b}$$

Both C_1 and C_2 can be expressed in terms of the absorption constant α, namely

$$C_1 = (\alpha L/2)[1+(\Delta\omega/\gamma_\perp)^2]^{-1} \tag{9a}$$

$$C_2 = (\alpha L/2)[1+(\Delta\omega/\gamma_\perp)^2]^{-1}\Delta\omega/\gamma_\perp. \tag{9b}$$

Equations (8*b*) and (9*b*) show that C_2 is proportional to $\Delta\omega$.

In the case of pure absorptive bistability, i.e. $\Delta\omega = 0$ (resonant case), $C_2 = 0$. Therefore C_2 measures the dispersive contribution to bistability. Since $\Delta\omega > \gamma_\perp$ implies $C_2 > C_1$, the case $C_1/C_2 \to 0$ characterises pure dispersive bistability. On the contrary

C_1 measures the absorptive contribution to bistability. Equation (6) relates the incoming and transmitted field amplitudes through a complex nonlinear susceptibility. In order to discuss this relation it is useful to go over to field intensities, namely

$$I_I = I_T\{[1+2C_1/(1+I_T)]^2 + 4C_2^2/(1+I_T)^2\}. \tag{10}$$

Since we are mainly interested in the properties of pure dispersive bistability, we first discuss (10) in the limit $C_1 = 0$. In this limit (10) shows the following characteristic behaviour: for $C_2 < C_{2th}$, I_I is a single-valued function of I_T, whereas for $C_2 > C_{2th}$ an interval of I_T values exists for which I_I is a multi-valued function of I_T. This behaviour of I_I versus I_T is characteristic of all descriptions of optical bistability. The threshold for bistable behaviour is given by

$$C_2^{\text{disp}} > 3\sqrt{3}/2 \equiv C_{2th}. \tag{11}$$

This condition, when compared with that which holds for pure absorptive bistability, i.e. $C_1^{\text{abs}} > 4$, shows that the threshold for pure dispersive bistability is lower than that for the pure absorptive case.

When the corresponding threshold intensities of the incoming fields are compared, they give the ratio

$$I_I^{\text{disp}}/I_I^{\text{abs}} = 8/27. \tag{12}$$

Hence the threshold intensity for the dispersive case is only ~30% of that required in the absorptive case. Equations (11) and (12) allow a quantitative explanation of the fact that experiments on optical bistability mainly deal with dispersive systems: threshold condition and intensity are smaller in the dispersive case. Indeed, from (10) with $C_1 \neq 0$, $C_2 \neq 0$, it can be shown that if $C_1 < 4$ the pure dispersive case is approached, whereas if $C_1 > 4$ the absorptive behaviour dominates.

We can further illustrate this point by considering (6) in the limiting case $C_1 = 0$ and $I_T < 1$. In this limit, the relation between the intensities becomes

$$I_I = I_T[1 + 4C_2^2(1 - I_T)^2]. \tag{13}$$

Equation (13) is identical to the phenomenological expression proposed by Gibbs *et al* (1976, 1978). In order to show this, we only have to normalise the intensities and identify $R \sim 1$, $4C_2^2 = \phi_0^3/\beta$ in Gibbs *et al*'s equation. The term $4C_2^2(1-I_T)^2$ corresponds to the square of the phase ϕ of the field. In fact, from (1) and (6), in the same approximation required for (13) it follows that

$$\phi \sim 2C_2(1 - I_T). \tag{14}$$

As is well known, (13) describes the bistable behaviour of an arbitrary dispersive medium in a cavity. The same approximations in the case of pure absorptive bistability lead to the equation

$$I_I = I_T[1 + 2C_1(1 - I_T)]^2 \tag{15}$$

which does not show any bistable behaviour.

Bistable behaviour can also be shown using different nonlinear dispersive models, e.g. nonlinear oscillators with fourth-order anharmonicity. In this case the relations (13) and (14) for intensities and phase still hold. The dispersive constant C_2 has of course to be redefined in terms of the parameters of the system. This result seems to indicate that bistable behaviour is a general property of nonlinear systems and does not depend on the particular model chosen for the medium.

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